REVISION 3 (Solutions)

Year 11 Examination

Question/Answer Booklet

MATHEMATICS METHODS UNITS 1 AND 2

Section One: Calculator-free

Time allowed for this section

Reading time before commencing work: five minutes
Working time for this section: fifty minutes

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer Booklet
Formula Sheet

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: nil

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Section One: Calculator-free **35% (51 Marks)**

This section has **nine (9)** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time for this section is 50 minutes.

Question 1 **(4 marks)**

A box contains a total of 500 marker and highlighter pens of various colours, as shown in the table. Some of the marker pens are permanent and the rest are non-permanent.

	Colour			
Type of pen	Black	Yellow	Pink	Green
Permanent marker	55	83	40	24
Non-permanent marker	45	67	24	12
Highlighter	0	50	46	54

A pen is selected at random from the box. Determine the probability that it is

(a) a yellow pen. (1 mark)

Solution
$\frac{83+67+50}{500}=\frac{200}{500}=\frac{2}{5}$
Specific behaviours
✓ correct probability

(b) a marker pen. (1 mark)

Solution
$\frac{500-(50+46+54)}{500}=\frac{350}{500}=\frac{7}{10}$
Specific behaviours
✓ correct probability

(c) a yellow pen or a marker pen. (1 mark)

Solution
$\frac{200+350-83-67}{500}=\frac{400}{500}=\frac{4}{5}$
Specific behaviours
✓ correct probability

(d) a green pen, given that it is a highlighter. (1 mark)

Solution
$\frac{54}{50+46+54}=\frac{54}{150}=\frac{9}{25}\ (=0.36)$
Specific behaviours
✓ correct probability

Question 2 **(6 marks)**

(a) Evaluate $\frac{m^{0.5}}{n^2}$ when $m = 4 \times 10^6$ and $n = 5 \times 10^2$, writing your answer without the use of scientific notation. (3 marks)

Solution
$\frac{m^{0.5}}{n^2} = \frac{\sqrt{4} \times 10^3}{25 \times 10^4}$ $= \frac{2}{25} \times \frac{1}{10}$ $= \frac{1}{125}$
Specific behaviours
✓ simplifies $m^{0.5}$ ✓ simplifies n^2 ✓ correct value

(b) Determine the value of x when $4^x = 32\sqrt{2}$. (3 marks)

Solution
$2^{2x} = 2^5 \times 2^{\frac{1}{2}}$ $= 2^{\frac{11}{2}}$ $2x = \frac{11}{2} \Rightarrow x = \frac{11}{4}$
Specific behaviours
✓ LHS as power of 2 ✓ RHS as power of 2 ✓ equates indices and solves

Question 3 **(7 marks)**

Solve each equation below for x.

(a) $\dfrac{3x}{x-5} = \dfrac{2}{3}$. (2 marks)

Solution
$9x = 2x - 10$ $7x = -10$ $x = -\dfrac{10}{7}$
Specific behaviours
✓ cross multiplies ✓ correct solution

(b) $(x+3)(x-3) = 8x$. (3 marks)

Solution
$x^2 - 9 = 8x$ $x^2 - 8x - 9 = 0$ $(x+1)(x-9) = 0$ $x = -1, \quad x = 9$
Specific behaviours
✓ expands and equates to zero ✓ factorises ✓ correct solutions

(c) $\sqrt{2}\sin x + 1 = 0,\ 0° \leq x \leq 360°$. (2 marks)

Solution
$\sin x = -\dfrac{1}{\sqrt{2}}$ $x = 225°, \quad x = 315°$
Specific behaviours
✓ one correct solution ✓ both correct solutions

Question 4 **(6 marks)**

(a) A and B are independent events such that $P(A) = \frac{2}{3}$ and $P(B) = \frac{1}{4}$. Determine

(i) $P(A \cap B)$. (1 mark)

Solution
$\frac{2}{3} \times \frac{1}{4} = \frac{1}{6}$
Specific behaviours
✓ calculates probability

(ii) $P(B|A)$. (1 mark)

Solution
$\frac{1}{4}$
Specific behaviours
✓ writes $P(B)$

(iii) $P(A \cup B)$. (2 marks)

Solution
$\frac{2}{3} + \frac{1}{4} - \frac{1}{6} = \frac{8+3-2}{12} = \frac{9}{12} = \frac{3}{4}$
Specific behaviours
✓ uses probability law ✓ calculates probability

(b) A number is selected at random from the set of positive integers. Event P occurs when the number is odd, event Q occurs when the number is a multiple of five and event R occurs when the number is a perfect square. Determine the smallest number that belongs to the following sets:

(i) $\bar{P} \cap (Q \cup R)$. (1 mark)

Solution
Even and either MF or PS: **4**
Specific behaviours
✓ writes number

(ii) $\bar{P} \cap Q \cap R$. (1 mark)

Solution
Even and MF and PS: **100**
Specific behaviours
✓ writes number

Question 5 **(4 marks)**

(a) Expand $(x+1)^4$. (2 marks)

Solution
$(x+1)^4 = (1)(x)^4(1)^0 + (4)(x)^3(1)^1 + (6)(x)^2(1)^2 + (4)(x)^1(1)^3 + (1)(x)^0(1)^4$ $= x^4 + 4x^3 + 6x^2 + 4x + 1$
Specific behaviours
✓ correct method ✓ correct expansion

(b) Determine the gradient of the curve $y = (x+1)^4$ at the point $(-2, 1)$. (2 marks)

Solution
$\frac{dy}{dx} = 4x^3 + 12x^2 + 12x + 4$ $x = -2 \Rightarrow \frac{dy}{dx} = 4(-8) + 12(4) + 12(-2) + 4 = -4$
Specific behaviours
✓ differentiates expression from (a) ✓ evaluates gradient

Question 6 **(5 marks)**

Determine the gradient of the curve $y = x^2 + 4x - 45$ at the point(s) where it crosses the x-axis.

Solution
$(x-5)(x+9) = 0$ $x = 5, x = -9$ $\frac{dy}{dx} = 2x + 4$ $x = 5, \frac{dy}{dx} = 14$ $x = -9, \frac{dy}{dx} = -14$ At $(5, 0)$ gradient is 14 and at $(-9, 0)$ gradient is -14.
Specific behaviours
✓ factorises quadratic ✓ determines roots ✓ derivative of quadratic ✓ one point and gradient ✓ second point and gradient

Question 7 **(7 marks)**

(a) Determine the coefficient of the n^3 term in the expansion of $(3n-1)^5$. (3 marks)

Solution
$(3n-1)^5 = \cdots + \binom{5}{2}(3n)^3(-1)^2 + \cdots$ Coefficient is $10 \times 27 \times 1 = 270$
Specific behaviours
✓ identifies correct term ✓ uses $\binom{5}{2}$ in expansion ✓ correct coefficient

(b) Consider the equation $x^3 - 7x^2 + 36 = 0$.

(i) Show that $x = 3$ is a solution of the equation. (1 mark)

Solution
$27 - 63 + 36 = 63 - 63 = 0$
Specific behaviours
✓ substitutes and expands

(ii) Determine all other solutions. (3 marks)

Solution
$x^3 - 7x^2 + 36 = (x-3)(x^2 - 4x - 12)$ $= (x-3)(x+2)(x-6)$ Other solutions: $x = -2, x = 6$
Specific behaviours
✓ determines quadratic factor ✓ factorises cubic ✓ states other two solutions

Question 8 **(4 marks)**

The line segment between the points $A(3,2)$ and $B(3,-4)$ is the diameter of a circle.

Determine the equation of circle in the form $x^2 + ax + y^2 + by = c$, where a, b and c are constants.

Solution
Centre: $\left(3, \frac{2-4}{2}\right) = (3,-1)$ Radius: $r = 2 - -1 = 3$ Equation: $(x-3)^2 + (y+1)^2 = 3^2$ $x^2 - 6x + 9 + y^2 + 2y + 1 = 9$ $x^2 - 6x + y^2 + 2y = -1$
Specific behaviours
✓ centre ✓ radius ✓ factored equation ✓ correct equation

Question 9 **(8 marks)**

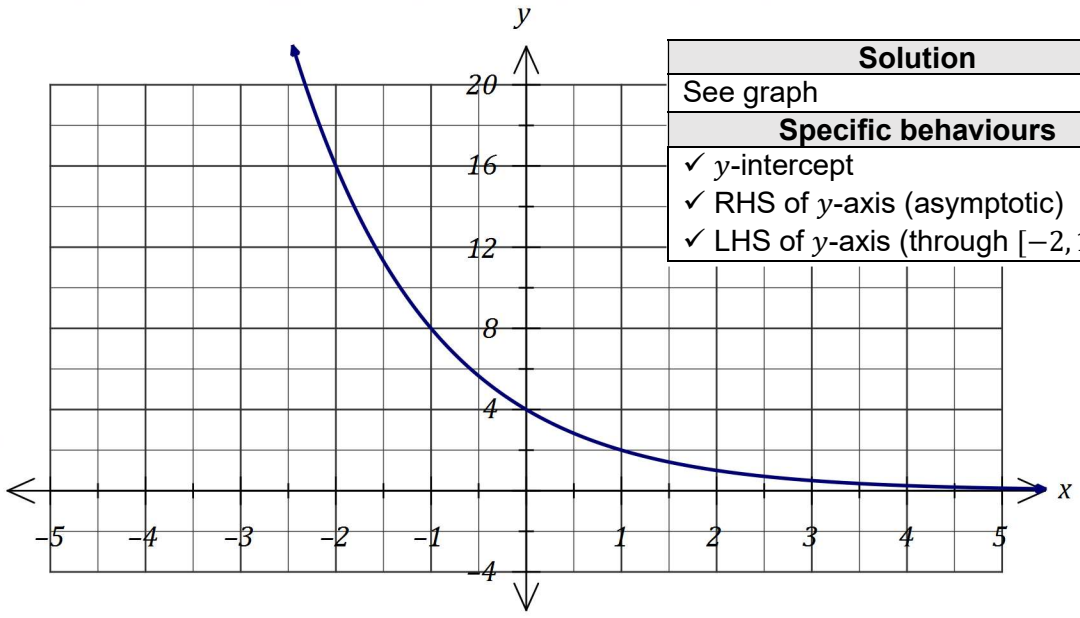
(a) Simplify $(2t - 5\sqrt{t})(2t + 5\sqrt{t})$. (2 marks)

Solution
$(2t - 5\sqrt{t})(2t + 5\sqrt{t}) = (2t)^2 - \left(5\sqrt{t}\right)^2$ $= 4t^2 - 25t$
Specific behaviours
✓ indicates use of difference of squares ✓ correct simplification

(b) Solve the equation $9^{2x} = \dfrac{\sqrt{3}}{81}$ for x. (3 marks)

Solution
$(3^2)^{2x} = 3^{0.5} \times 3^{-4}$ $3^{4x} = 3^{-3.5}$ $4x = -3.5$ $x = -0.875 = -\dfrac{7}{8}$
Specific behaviours
✓ writes 9 and 81 as powers of 3 ✓ simplifies RHS ✓ correct solution

(c) Sketch the graph of $y = 2^{(2-x)}$ on the axes below. (3 marks)



Solution
See graph
Specific behaviours
✓ y-intercept ✓ RHS of y-axis (asymptotic) ✓ LHS of y-axis (through $[-2, 16]$)